South East Asian J. of Mathematics and Mathematical Sciences
Vol. 22, No. 1 (2026), pp. 125-142

DOI: 10.56827/SEAJMMS.2026.2201.9 **ISSN (Online): 2582-0850**

ISSN (Print): 0972-7752

CERTAIN INTEGRAL TRANSFORM OF PATHWAY FRACTIONAL INTEGRAL OPERATOR ASSOCIATED WITH THE PRODUCT OF (p, q)-EXTENDED BESSEL FUNCTION AND GENERALIZED k-STRUVE FUNCTION

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(**Received:** *Sep. 07, 2025* **Accepted:** *Mar. 13, 2026* **Published:** *Apr. 30, 2026)*

Abstract: Motivated by a recent work on Pathway fractional integral operator associated with the generalized k-Struve function,this paper establishes five theorems by using Pathway fractional integral operator involving the product of the (p, q)-Extended Bessel function and Generalized k-Struve function, supported by several auxiliary lemma. The results are expressed in terms of the ${}_{r+k}F_{s+k}$ and generalized k-Wright function ${}_r\psi_s{}^k$.Some new and known results are also obtained in special cases of main results. Then, their certain Integral transforms including Jafari transform via Pathway Fractional Integral formulas Involving Product of (p, q)-Extended Bessel function and Generalized k-Struve Function.

Keywords and Phrases: Pathway Fractional Integral Operator, (p, q)-Extended Bessel function and Generalized k-Struve function, (p, q)-Extended generalized hypergeometric function and Generalized Wright function, Jafari transform.

2020 Mathematics Subject Classification: Primary 26A33, Secondary 33C45, 33E12, 44A20.

1. Introduction

The fractional calculus is a field of applied mathematics that deals with the fractional derivative and integrals of arbitrary orders.Fractional calculus operators are studied extensively due to their importance in applied problems of science and engineering. Special functions such as the Bessel function and the generalized k-Struve function play a significant role in solving many problems related to wave propagation, heat conduction, and mathematical modeling. In recent years, several researchers have investigated different integral transforms associated with special functions. Motivated by these developments, the present paper studies the pathway fractional integral operator associated with the product of the (p,q)-extended Bessel function and the generalized k-Struve function. In this paper we will establish integral transform of new fractional calculus formulas involving some various special functions. The Pathway fractional integral operator involving some special functions defined (see also [26], [19], [20]) thus. As a new member of the family of general integral transforms. Consequently, numerous researchers and scientists have been consistently engaged in this evolving field, with the aim exploring and under standing the implications of these changes [18].

The Riemann-Liouville Fractional integral Operator. Let $\phi(z) \in L(m,n)$, $\lambda \in C, \Re(\lambda) > 0$,then the Riemann-Liouville Fractional integral operator is given by as

$$\left(I_{0_+}^{\lambda}\phi\right) z = \frac{1}{\Gamma(\lambda)} \int_0^z (z-u)^{\lambda-1}\phi(u)du, \tag{1.1}$$

where $\Re(\lambda)$denotes the real part of λ. We refer to Kiryakova et.al. [13].

Pathway Fractional Integral Operators. The pathway modal is developed by Mathai [15] and studied further by Mathai and Haubold (see also [16], [17]) and introduced by Pathway Fractional Integral operators in Nair [23].

Definition 1.1. *Let $\phi(z) \in L(m,n), \lambda \in C, \Re(\lambda) > 0, m > 0$ and let us take a "pathway parameter" $\lambda < 1$. Then the pathway fractional integral operator, as extension of* (1.1)*, is defined by as follows:*

$$\left(P_{0_+}^{(\zeta,\lambda)}\phi\right) z = x^{\zeta} \int_0^{\left[\frac{z}{m(1-\lambda)}\right]} \left[1 - \frac{m(1-\lambda)u}{z}\right]^{\frac{\zeta}{1-\lambda}} \phi(u)du. \tag{1.2}$$

For a real scalar λ, the pathway modal for scalar random variables is represented by the following probability density functions:

$$\phi(z) = c|z|^{t-1}\left[1 - m(1-\lambda) \mid z \mid^{h}\right]^{\frac{q}{1-\lambda}}, \tag{1.3}$$

provided that $-\infty < z < \infty; h > 0; g \geq 0; \left[1 - m(1-\lambda)|z|^h\right] > 0$ and $t > 0$, where is l the normalizing constant and λ is called the pathway parameter [21].

For more details on the pathway model, the reader is invited to consider references (see also [2], [14], [5], [1], [27], [24]).

Remark 1.1. *When* $\lambda \to 1_-$, $\left[1 - \frac{m(1-\lambda)k}{z}\right]^{\frac{\zeta}{(1-\lambda)}} \to e^{-\frac{m\zeta}{z}k}$. *Then, pathway fractional integral operator* (1.2), *we get*

$$\left(P_{0_-}^{(\zeta,1)}\phi\right)z = x^{\zeta}\int_0^{\infty} e^{-\frac{m\zeta}{z}k}\phi(k)dk = x^{\zeta}L\left[\phi(\frac{m\zeta}{z})\right], \tag{1.4}$$

that is, it reduces to the Laplace integral transform of f with parameter $\frac{m\zeta}{z}$*:*

$$L\left[\phi(z)\right] = \int_0^{\infty} e^{-zk}\phi(k)dk. \tag{1.5}$$

Remark 1.2. *When* $\lambda = 0, m = 1$, *then replacing* ζ *by* $\zeta - 1$ *in* (1.2), *we get*

$$\int_0^{z}(z-k)^{\zeta-1}\phi(k)dk = \Gamma(\zeta)\left(I_{0_+}^{\zeta}\phi\right)z \tag{1.6}$$

and reduces to the left-sided Riemann-Liouville fractional integral $I_{0_+}^{\zeta}$ *in* (1.1).

Generalized k-Struve function. Nisar et. al. [22] defined by the generalized k-Struve function in the following manner:

$$S_{v,c}^{k}(t) = \sum_{n=0}^{\infty}\frac{(-c)^n}{\Gamma_k\left(nk+v+\frac{3k}{2}\right)\Gamma\left(n+\frac{3}{2}\right)}\left(\frac{t}{2}\right)^{2n+\frac{v}{k}+1} \quad \left(k\in\Re^+; c\in\Re; v>-1\right) \tag{1.7}$$

where the generalized k-Struve function $S_{v,c}^{k}(t)$ convergence for all complex values of t for details by Khan et. al. [12].

Proposition 1.1. *The k-Gamma function defined by Daiz et. al.* [8] *in the following manner:*

$$\Gamma_k(z) = \int_0^{\infty} t^{z-1}e^{-\frac{t^k}{k}}dt, z\in\mathbb{C}, k>0 \tag{1.8}$$

Proposition 1.2. *If* $z \in \mathbb{C}$ *and* $k \in \Re$, *then the following identity is true*

$$\Gamma_k\left(z+k\right) = z\Gamma_k\left(z\right) \tag{1.9}$$

and

$$\Gamma_k(z) = k^{\frac{z}{k}-1}\Gamma\left(\frac{z}{k}\right) \tag{1.10}$$

Remark 1.3. *If we taking $k \to 1$ and $c = 1$ in Eq.* (1.7) *reduces to well-known Struve function of order v defined by* [4] *as*

$$H_v(z) = \sum_{r=0}^{\infty} \frac{(-1)^r}{\Gamma\left(r+v+\frac{3}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{z}{2}\right)^{2r+v+1} \tag{1.11}$$

Remark 1.4. *If we taking $c = -1$ and $k = 1$ in* (1.7), *then we get modified Struve function of order v defined by* [4] *as*

$$L_v(z) = \sum_{r=0}^{\infty} \frac{1}{\Gamma\left(r+v+\frac{3}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{z}{2}\right)^{2r+v+1} \tag{1.12}$$

The relation between generalized k-Struve function, cosine and sine function is as follows ([22], pp.1270, eq.(10), eq.(11), eq.(12) and eq.(13)):

$$1-\cos\left(\frac{\varpi z}{\sqrt{k}}\right) = \frac{\varpi}{k}\sqrt{\left(\frac{\pi z}{2}\right)}S^{k}_{\frac{k}{2},\varpi^2}(z) \tag{1.13}$$

$$\cosh\left(\frac{\varpi z}{\sqrt{k}}\right)-1 = \frac{\varpi}{k}\sqrt{\left(\frac{\pi z}{2}\right)}S^{k}_{\frac{k}{2},-\varpi^2}(z) \tag{1.14}$$

$$\sin\left(\frac{\varpi z}{\sqrt{k}}\right) = \varpi\sqrt{\left(\frac{\pi z}{2k}\right)}S^{k}_{-\frac{k}{2},\varpi^2}(z) \tag{1.15}$$

$$\sinh\left(\frac{\varpi z}{\sqrt{k}}\right) = \varpi\sqrt{\left(\frac{\pi z}{2k}\right)}S^{k}_{-\frac{k}{2},-\varpi^2}(z) \tag{1.16}$$

For further study about Struve functions and properties, the interesting can be refereed reader to (see also [25], [31]).

(p, q) **- Extended Bessel function.** The (p, q)-extended Bessel function $J_{\nu,p,q}(z)$ of the first kind of order ν is defined as follows [28].

$$J_{\nu,p,q}(z) = \frac{\sqrt{\pi}}{\Gamma(\nu+1)}\sum_{m=0}^{\infty}\frac{(-1)^m B(m+\frac{1}{2},\nu+\frac{1}{2};p,q)}{m!\Gamma(m+\frac{1}{2})B(\frac{1}{2},\nu+\frac{1}{2})}\left(\frac{z}{2}\right)^{2m+\nu} \tag{1.17}$$

Where $min\{\Re(p), \Re(q)\} \geq 0$ and $\Re(\nu) > -1$ when $p = q = 0$.

Proposition 1.3. *The (x, y)-extended Beta function defined by Choi et. al. [7] in the following manner as:*

$$B\left(u, w; x, y\right) = \int_0^1 t^{u-1}(1-t)^{w-1}e^{\left(-\frac{x}{t}-\frac{y}{1-t}\right)} \tag{1.18}$$

$$(min\{\Re(u),\Re(w)\}>0; min\,\{\Re(x),\Re(y)\}\geq 0)$$

It should be remarked here that the existing literature on the subject contains much more general extensions of the classical Beta function, especially in the case when $x=y$.
For $x=y=1$, the (x,y)-extended Bessel function of the first kind $J_{\nu,x,y}(z)$ and the (x,y)-extended Beta function $B(u,w;x.y)$ reduces Bessel function $J_\nu(z)$ of the first kind and the classical Beta function $B(x,y)$, respectively.

(p,q)**-Extended Gauss hypergeometric function.** The (p,q)-extended Gauss hypergeometric function $F_{p,q}$ is defined as follows [7]:

$$F_{p,q}(u,v;w;z)=\sum_{n=0}^{\infty}(u)_n\frac{B(v+n,w-v;p,q)}{B(v,w-v)}\frac{z^n}{n!} \tag{1.19}$$

where $|z|<1$ and $\Re(w)>\Re(v)>0$.

(p,q)**-Extended Generalized hypergeometric function.** The (p,q)-extended generalized hypergeometric functions is defined as follows (see also [11], pp.621, eq.(2.8)):

$${}_{r+k}F_{s+k}\left[\begin{array}{c} a_1,....,a_r,A_i,....,A_k; \\ c_i,....c_s,C_1,....,C_k;\end{array} z;p,q\right]=\sum_{n\geq 0}^{\infty}\frac{\prod_{j=1}^{r}(a_j)_n}{\prod_{j=1}^{s}(c_j)_n}\prod_{j=1}^{k}\frac{B(A_j+n,C_j-A_j;p,q)}{B(A_j,C_j-A_j)}\frac{z^n}{n!} \tag{1.20}$$

The special case of the (p,q)-extended generalized hypergeometric function ; when e.g. $k=r=s+1=1$; and *a fortiori* $a_1=a, A_1=b, C_1=c$ becomes the already known (p,q)-extended Gaussian hypergeometric function (see also [6], [7]).

Generalized Wright Function and k- Wright Function. The generalized Wright function defined by ([29], [30]) in the following manner as:

$$\begin{aligned}
{}_r\psi_s(x) &={}_r\psi_s\left[\begin{array}{c}(a_i,C_i)_{1,r}; \\ (b_i,D_i)_{1,s};\end{array} x\right] \\
&={}_r\psi_s\left[\begin{array}{c}(a_1,C_1),(a_2,C_2),...,(a_r,C_r); \\ (b_1,D_1),(b_2,D_2),...,(b_s,D_s);\end{array} x\right] \\
&=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{r}\Gamma(a_i+nC_i)}{\prod_{j=1}^{s}\Gamma(b_j+nD_j)}\frac{x^n}{n!}, x\in\mathbb{C} \\
&=\sum_{n=0}^{\infty}\frac{\Gamma(a_1+nC_1)\Gamma(a_2+nC_2).....\Gamma(a_p+nC_p)}{\Gamma(b_1+nD_1)\Gamma(b_2+nD_2).....\Gamma(b_q+nD_q)}\frac{x^n}{n!}, x\in\mathbb{C}
\end{aligned} \tag{1.21}$$

Gehlot et. al. [9] defined by the generalized from of the above Wright function in Eq.(1.21), named as generalized K-Wright function is defined as follows:

$$
\begin{aligned}
{}_r\psi_s^k(x) &= {}_r\psi_s^k\left[\begin{array}{l}(a_i, C_i)_{1,r};\\ (b_i, D_i)_{1,s};\end{array} x\right]\\
&= {}_r\psi_s^k\left[\begin{array}{l}(a_1, C_1), (a_2, C_2), ..., (a_r, C_r);\\ (b_1, D_1), (b_2, D_2), ..., (b_s, D_s);\end{array} x\right]\\
&= \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{r} \Gamma_k(a_i + nC_i)}{\prod_{j=1}^{s} \Gamma_k(b_j + nD_j)} \frac{x^n}{n!}, x \in \mathbb{C}\\
&= \sum_{n=0}^{\infty} \frac{\Gamma_k(a_1 + nC_1)\Gamma_k(a_2 + nC_2).....\Gamma_k(a_r + nC_r)}{\Gamma_k(b_1 + nD_1)\Gamma_k(b_2 + nD_2).....\Gamma_k(b_x + nD_s)} \frac{x^n}{n!}, x \in \mathbb{C} \qquad (1.22)
\end{aligned}
$$

Integral Transforms. We introduce basic definition of the certain integral transforms as Jafari transform.

Definition 1.2. *The Jafari transform $T(\sigma)$ of $\nu(t)$ is defined as follows* [10]*:*

$$T\{\nu(t), \sigma\} = T(\sigma) = h(\sigma) \int_0^{\infty} \nu(t) e^{-g(\sigma)t} dt = R\left(h(\sigma), g(\sigma)\right), \qquad (1.23)$$

where $\nu(t)$ be a integrable function defined for $t \geq 0, h(\sigma) \neq 0$ and $g(\sigma)$ and integral exists for some $g(\sigma)$.

Some Required Lemma. The required lemmas defined by Baleanu et.al. [3] in the following manner as:

Lemma. *Let $\zeta \in \mathbb{C}, \Re(\zeta) > 0, \tau \in \mathbb{C}$ and $\lambda < 1$.If $\Re(\tau) > 0$ and $\Re(\frac{\zeta}{1-\lambda}) > -1$, then*

$$\left(P_{0_+}^{(\zeta,\lambda)}[t^{\tau-1}]\right)(x) = \frac{x^{\zeta+\tau}}{[m(1-\lambda)]^{\tau}} \frac{\Gamma(\tau)\Gamma(1+\frac{\zeta}{1-\lambda})}{\Gamma(1+\frac{\zeta}{1-\lambda}+\tau)} \qquad (1.24)$$

2. Pathway Fractional Integration Involving the Product of (p, q)-Extended Bessel function and Generalized k-Struve function

In this section, we present certain pathway fractional integration formulas involving the product of (p, q)-Extended Bessel function and Generalized k-Struve function by using Pathway fractional integral operators are established here as some auxiliary five theorems.

Theorem 2.1. Let the parametric constraints $\zeta, \tau, v, c \in \mathbb{C}$ and $\lambda < 1$ be such that $\Re(\zeta) > 0, \Re(\tau) > 0, v > \frac{-3k}{2}$ and $\Re(\frac{\tau}{1-\lambda}) > -1$, then the following result is given by:

$$\left(P_{0_+}^{(\zeta,\lambda)}\left[t^{\tau-1}J_{\nu,p,q}(t^{\mu})S_{v,c}^{k}(t)\right]\right)(x)=\frac{x^{\zeta}}{\Gamma(\nu+1)}\frac{1}{k^{\frac{v}{k}+\frac{1}{2}}}\frac{1}{2^{\frac{v}{k}+1+\nu}}\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)$$

$$\left(\frac{x}{m(1-\lambda)}\right)^{\tau+\frac{v}{k}+\mu\nu+1}\times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle|-\frac{x^{2\mu}}{4m^{2\mu}(1-\lambda)^{2\mu}};p,q\right]$$

$$\times{}_2\psi_3\left[\begin{array}{ccc}(\tau+\frac{v}{k}+2n\mu+\mu\nu+1,2),(1,1) & ; & -\frac{x^2c}{4km^2(1-\lambda)^2}\\ (\tau+\frac{v}{k}+2n\mu+\mu\nu+\frac{\zeta}{1-\lambda}+2,2),(\frac{v}{k}+\frac{3}{2},1),(\frac{3}{2},1) & ; & \end{array}\right] \tag{2.1}$$

Proof. We denotes the left-hand sides of theorem (2.1) by Δ,

$$\Delta=\left(P_{0_+}^{(\zeta,\lambda)}\left[t^{\tau-1}J_{\nu,p,q}(t^{\mu})S_{v,c}^{k}(t)\right]\right)(x) \tag{2.2}$$

Using formula (1.7) and (1.17) in the right -hand side of Eq.(2.2) then change the order of the Pathway fractional integration and summation, we find that

$$\begin{aligned}\Delta=&\frac{\sqrt{\pi}}{\Gamma(\nu+1)}\sum_{n=0}^{\infty}\frac{(-1)^nB(n+\frac{1}{2},\nu+\frac{1}{2};p,q)}{n!\Gamma(n+\frac{1}{2})B(\frac{1}{2},\nu+\frac{1}{2})}\left(\frac{1}{2}\right)^{2n+\nu}\\ &\times\sum_{r=0}^{\infty}\frac{(-c)^r}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\\ &\times\left(\frac{1}{2}\right)^{2r+\frac{v}{k}+1}\times\left(P_{0_+}^{(\zeta,\lambda)}\left[t^{\tau+2n\mu+\mu\nu+\frac{v}{k}+2r+1-1}\right]\right)(x)\end{aligned} \tag{2.3}$$

On applying lemma (1.24) in Eq.(2.3),we get

$$\begin{aligned}\Delta=&x^{\zeta+\tau+\mu\nu+\frac{v}{k}+1}\left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1}\times\frac{1}{[m(1-\lambda)]^{\tau+\mu\nu+\frac{v}{k}+1}}\times\frac{\sqrt{\pi}}{\Gamma(\nu+1)}\\ &\times\sum_{n=0}^{\infty}\frac{B(n+\frac{1}{2},\nu+\frac{1}{2};p,q)}{n!\Gamma(n+\frac{1}{2})B(\frac{1}{2},\nu+\frac{1}{2})}\left(-\frac{x^{2\mu}}{4m^{2\mu}(1-\lambda)^{2\mu}}\right)^n\\ &\times\sum_{r=0}^{\infty}\frac{1}{\Gamma_k\left(rk+v+\frac{3k}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{-cx^2}{4m^2(1-\lambda)^2}\right)^r\\ &\times\frac{\Gamma\left(\tau+\frac{v}{k}+2n\mu+\mu\nu+2r+1\right)}{\Gamma\left(1+\frac{\zeta}{1-\lambda}+\tau+\frac{v}{k}+2n\mu+\mu\nu+2r+1\right)}\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\end{aligned} \tag{2.4}$$

Now using Eq. (1.10) in (2.4), we get

$$\begin{aligned}\Delta =& x^{\zeta+\tau+\mu\nu+\frac{v}{k}+1}\frac{1}{k^{\frac{v}{k}+\frac{1}{2}}}\left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1} \times \frac{1}{[m(1-\lambda)]^{\tau+\mu\nu+\frac{v}{k}+1}}\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\\ &\times\frac{\sqrt{\pi}}{\Gamma(\nu+1)}\times\frac{1}{\sqrt{\pi}}\times\sum_{n=0}^{\infty}\frac{B(n+\frac{1}{2},\nu+\frac{1}{2};p,q)}{n!\left(\frac{1}{2}\right)_n B(\frac{1}{2},\nu+\frac{1}{2})}\left(-\frac{x^{2\mu}}{4m^{2\mu}(1-\lambda)^{2\mu}}\right)^n\\ &\times\sum_{r=0}^{\infty}\frac{1}{\Gamma\left(r+\frac{v}{k}+\frac{3}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\left(\frac{-cx^2}{4km^2(1-\lambda)^2}\right)^r\\ &\times\frac{\Gamma\left(\tau+\frac{v}{k}+2n\mu+\mu\nu+2r+1\right)}{\Gamma\left(1+\frac{\zeta}{1-\lambda}+\tau+\frac{v}{k}+2n\mu+\mu\nu+2r+1\right)}\end{aligned}\tag{2.5}$$

Now using Eq.(1.20) in Eq.(2.5) ,then we get

$$\begin{aligned}\Delta =& x^{\zeta+\tau+\mu\nu+\frac{v}{k}+1}\times\frac{1}{k^{\frac{v}{k}+\frac{1}{2}}}\left(\frac{1}{2}\right)^{\nu+\frac{v}{k}+1} \times \frac{1}{[m(1-\lambda)]^{\tau+\mu\nu+\frac{v}{k}+1}}\times\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\\ &\times\frac{1}{\Gamma(\nu+1)}\times {}_1F_2\left[\begin{matrix}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{matrix}\middle| -\frac{x^{2\mu}}{4m^{2\mu}(1-\lambda)^{2\mu}};p,q\right]\\ &\times\sum_{r=0}^{\infty}\frac{\Gamma(r+1)}{\Gamma\left(r+\frac{v}{k}+\frac{3}{2}\right)\Gamma\left(r+\frac{3}{2}\right)r!}\left(\frac{-cx^2}{4km^2(1-\lambda)^2}\right)^r\\ &\times\frac{\Gamma\left(\tau+\frac{v}{k}+2n\mu+\mu\nu+2r+1\right)}{\Gamma\left(1+\frac{\zeta}{1-\lambda}+\tau+\frac{v}{k}+2n\mu+\mu\nu+2r+1\right)}\end{aligned}\tag{2.6}$$

Using the definition of (1.22) in (2.6), we at once arrive at the desired result in (2.1).

Theorem 2.2. *Let the parametric constraints* $\zeta,\tau,v,c\in\mathbb{C}$ *and* $\lambda<1,|\frac{1}{t}|\leq 1$ *be such that* $\Re(\zeta)>0,\Re(\tau)>0,v>\frac{-3k}{2}$ *and* $\Re(\frac{\tau}{1-\lambda})>-1$ *,then the following result is given by:*

$$\begin{aligned}&\left(P_{0_+}^{(\zeta,\lambda)}\left[t^{\tau-1}J_{\nu,p,q}(\sigma t^{-\mu})\left(1-\cos\left(\frac{\varpi t}{\sqrt{k}}\right)\right)\right]\right)(x)=x^{\zeta}\frac{\varpi\sqrt{\pi}}{4k^2\Gamma(\nu+1)}\times\left(\frac{\sigma}{2}\right)^{\nu}\\ &\left(\frac{x}{[m(1-\lambda)]}\right)^{\tau-\mu\nu+2}\times\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\times {}_1F_2\left[\begin{matrix}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{matrix}\middle| -\frac{\sigma^2m^{2\mu}(1-\lambda)^{2\mu}}{4x^{2\mu}};p,q\right]\\ &\times {}_2\psi_3\left[\begin{matrix}(\tau-2n\mu-\mu\nu+2,2),(1,1) & ; \\ (\tau-2n\mu+\mu\nu-\frac{\zeta}{1-\lambda}+3,2),(2,1),(\frac{3}{2},1) & ;\end{matrix} -\frac{\varpi^2x^2}{4km^2(1-\lambda)^2}\right]\end{aligned}\tag{2.7}$$

Theorem 2.3. *Let the parametric constraints $\zeta, \tau, v, c \in \mathbb{C}$ and $\lambda < 1, |\frac{1}{t}| \leq 1$ be such that $\Re(\zeta) > 0, \Re(\tau) > 0, v > \frac{-3k}{2}$ and $\Re(\frac{\tau}{1-\lambda}) > -1$, then the solution of the following formula is given by :*

$$\left(P_{0_+}^{(\zeta,\lambda)}\left[t^{\tau-1}J_{\nu,p,q}(\sigma t^{-\mu})\left(\cosh\left(\frac{\varpi t}{\sqrt{k}}\right)-1\right)\right]\right)(x) = x^{\zeta}\frac{\varpi\sqrt{\pi}}{4k^2\Gamma(\nu+1)} \times \left(\frac{\sigma}{2}\right)^{\nu}$$

$$\times\left(\frac{x}{[m(1-\lambda)]}\right)^{\tau-\mu\nu+2}\Gamma\left(1+\frac{\zeta}{1-\lambda}\right) \times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2}, \nu+1\end{array}\middle| -\frac{\sigma^2 m^{2\mu}(1-\lambda)^{2\mu}}{4x^{2\mu}}; p, q\right]$$

$$\times {}_2\psi_3\left[\begin{array}{cc}(\tau-2n\mu-\mu\nu+2,2),(1,1) & ; \\ (\tau-2n\mu+\mu\nu-\frac{\zeta}{1-\lambda}+3,2),(2,1),(\frac{3}{2},1) & ;\end{array} \frac{\varpi^2 x^2}{4km^2(1-\lambda)^2}\right] \tag{2.8}$$

Theorem 2.4. *Let the parametric constraints$\zeta, \tau, v, c \in \mathbb{C}$ and $\lambda < 1, |\frac{1}{t}| \leq 1$ be such that $\Re(\zeta) > 0, \Re(\tau) > 0, v > \frac{-3k}{2}$ and $\Re(\frac{\tau}{1-\lambda}) > -1$, then the following result is given by:*

$$\left(P_{0_+}^{(\zeta,\lambda)}\left[t^{\tau-1}J_{\nu,p,q}(\sigma t^{-\mu}) sin\left(\frac{\varpi t}{\sqrt{k}}\right)\right]\right)(x) = x^{\zeta} \times \frac{\varpi}{2\Gamma(\nu+1)}\sqrt{\frac{\pi}{k}}\left(\frac{\sigma}{2}\right)^{\nu}$$

$$\times\left(\frac{x}{[m(1-\lambda)]}\right)^{\tau-\mu\nu+1} \times\Gamma\left(1+\frac{\zeta}{1-\lambda}\right) \times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2}, \nu+1\end{array}\middle| -\frac{\sigma^2 m^{2\mu}(1-\lambda)^{2\mu}}{4x^{2\mu}}; p, q\right]$$

$$\times {}_1\psi_2\left[\begin{array}{cc}(\tau-2n\mu-\mu\nu+1,2) & ; \\ (\tau-2n\mu-\mu\nu-\frac{\zeta}{1-\lambda}+2,2),(\frac{3}{2},1) & ;\end{array} -\frac{\varpi^2 x^2}{4km^2(1-\lambda)^2}\right] \tag{2.9}$$

Theorem 2.5. *Let the parametric constraints$\zeta, \tau, v, c \in \mathbb{C}$ and $\lambda < 1, |\frac{1}{t}| \leq 1$ be such that $\Re(\zeta) > 0, \Re(\tau) > 0, v > \frac{-3k}{2}$ and $\Re(\frac{\tau}{1-\lambda}) > -1$, then the following result is given by:*

$$\left(P_{0_+}^{(\zeta,\lambda)}\left[t^{\tau-1}J_{\nu,p,q}(\sigma t^{-\mu}) sinh\left(\frac{\varpi t}{\sqrt{k}}\right)\right]\right)(x) = x^{\zeta} \times \frac{\varpi}{2\Gamma(\nu+1)}\sqrt{\frac{\pi}{k}}\left(\frac{\sigma}{2}\right)^{\nu}$$

$$\times\left(\frac{x}{[m(1-\lambda)]}\right)^{\tau-\mu\nu+1}\Gamma\left(1+\frac{\zeta}{1-\lambda}\right) \times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2}, \nu+1\end{array}\middle| -\frac{\sigma^2 m^{2\mu}(1-\lambda)^{2\mu}}{4x^{2\mu}}; p, q\right]$$

$$\times {}_1\psi_2\left[\begin{array}{cc}(\tau-2n\mu-\mu\nu+1,2) & ; \\ (\tau-2n\mu-\mu\nu-\frac{\zeta}{1-\lambda}+2,2),(\frac{3}{2},1) & ;\end{array} \frac{\varpi^2 x^2}{4km^2(1-\lambda)^2}\right] \tag{2.10}$$

The proof of Theorem 2.2, 2.3, 2.4 and 2.5 are developed following the similar lines as to prove the Theorem 2.1 respectively in view of the definitions of generalized k-struve function and (p, q)-Extended Bessel function defined in (1.7) and (1.17).

3. Main Results

In this section, we apply the results involving previous sections using Jafari transform. The results are based on the definition of the Jafari transform given in (1.23).

Theorem 3.1. *Let the parametric constraints $\zeta, \tau, v, c \in \mathbb{C}, h(\sigma)$ and $g(\sigma)$ are positive real numbers but $(\sigma) \neq 0$ and $\lambda < 1$ be such that $\Re(\zeta) > 0, \Re(\tau) > 0, v > \frac{-3k}{2}$ and $\Re(\frac{\tau}{1-\lambda}) > -1$. The above condition ensures the convergence of the series representation and guarantees the validity of the obtained integral transform, then the below mentioned Jafari transformation holds true:*

$$T\left[P_{0_+}^{(\zeta,\lambda)}\{t^{\tau-1}J_{\nu,p,q}(t^\mu)S_{v,c}^k(tz)^\xi\}(x)\right] = \frac{h(\sigma)}{g(\sigma)}\frac{x^\zeta}{\Gamma(\nu+1)} \times \frac{1}{k^{\frac{v}{k}+\frac{1}{2}}} \times \frac{1}{2^{\frac{v}{k}+1+\nu}}$$

$$\Gamma\left(1+\frac{\zeta}{1-\lambda}\right) \times \left(\frac{x}{m(1-\lambda)}\right)^{\tau+\frac{v}{k}+\mu\nu+1} \times {}_1F_2\left[\begin{matrix}\frac{1}{2}\\ \frac{1}{2}, \nu+1\end{matrix}\,\middle|-\frac{x^{2\mu}}{4m^{2\mu}(1-\lambda)^{2\mu}}; p, q\right]$$

$$\times {}_3\psi_3\left[\begin{matrix}(\tau+\frac{v}{k}+2n\mu+\mu\nu+1,2),(1,1),(1,\xi) & ; \\ (\tau+\frac{v}{k}+2n\mu+\mu\nu+\frac{\zeta}{1-\lambda}+2,2),(\frac{v}{k}+\frac{3}{2},1),(\frac{3}{2},1) & ;\end{matrix} -\frac{x^{2+\xi}c}{4kg(\sigma)^\xi m^2(1-\lambda)^2}\right] \tag{3.1}$$

Proof. We denotes the left-hand sides of theorem (3.1) by Ω,

$$\Omega = T\left[P_{0_+}^{(\zeta,\lambda)}\{t^{\tau-1}J_{\nu,p,q}(t^\mu)S_{v,c}^k(tz)^\xi\}(x)\right] \tag{3.2}$$

Using definition of (1.23) in the right -hand side of Eq.(3.2), we find that

$$\Omega = h(\sigma)\int_0^\infty e^{-g(\sigma)z}\left[P_{0_+}^{(\zeta,\lambda)}\{t^{\tau-1}J_{\nu,p,q}(t^\mu)S_{v,c}^k(tz)^\xi\}(x)\right]dz \tag{3.3}$$

On applying the result (2.1) in Eq.(3.3), we get

$$\Omega = h(\sigma)\int_0^\infty e^{-g(\sigma)z} \times \frac{x^\zeta}{\Gamma(\nu+1)}\frac{1}{k^{\frac{v}{k}+\frac{1}{2}}}\frac{1}{2^{\frac{v}{k}+1+\nu}}\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)$$

$$\left(\frac{x}{m(1-\lambda)}\right)^{\tau+\frac{v}{k}+\mu\nu+1} \times {}_1F_2\left[\begin{matrix}\frac{1}{2}\\ \frac{1}{2}, \nu+1\end{matrix}\,\middle|-\frac{x^{2\mu}}{4m^{2\mu}(1-\lambda)^{2\mu}}; p, q\right]$$

$$\sum_{r=0}^\infty \frac{\Gamma(r+1)}{\Gamma\left(r+\frac{v}{k}+\frac{3}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\frac{\Gamma\left(\tau+2r+2-2n\mu-\mu\nu\right)}{\Gamma\left(1+\frac{\zeta}{1-\lambda}+\tau-2n\mu-\mu\nu+2r+2\right)}$$

$$\left(\frac{-cx^{2+\xi}}{4km^2(1-\lambda)^2}\right)^r z^{\xi r}dz \tag{3.4}$$

Now interchanging the order of integration and summation in Eq.(3.4), then we get

$$\begin{aligned}\Omega =& h(\sigma)\frac{x^{\zeta}}{\Gamma(\nu+1)} \times \frac{1}{k^{\frac{v}{k}+\frac{1}{2}}} \times \frac{1}{2^{\frac{v}{k}+1+\nu}}\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\\ &\times\left(\frac{x}{m(1-\lambda)}\right)^{\tau+\frac{v}{k}+\mu\nu+1} \times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\Bigg| -\frac{x^{2\mu}}{4m^{2\mu}(1-\lambda)^{2\mu}};p,q\right]\\ &\times\sum_{r=0}^{\infty}\frac{\Gamma(r+1)}{\Gamma\left(r+\frac{v}{k}+\frac{3}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\frac{\Gamma\left(\tau+2r+2-2n\mu-\mu\nu\right)}{\Gamma\left(1+\frac{\zeta}{1-\lambda}+\tau-2n\mu-\mu\nu+2r+2\right)r!}\\ &\times\left(\frac{-cx^{2+\xi}}{4km^2(1-\lambda)^2}\right)^{r}\times\int_0^{\infty}e^{-sz}z^{1+\xi r-1}dz \end{aligned} \tag{3.5}$$

$$\begin{aligned}\Omega =& \frac{h(\sigma)}{g(\sigma)}\frac{x^{\zeta}}{s\Gamma(\nu+1)} \times \frac{1}{k^{\frac{v}{k}+\frac{1}{2}}} \times \frac{1}{2^{\frac{v}{k}+1+\nu}}\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\\ &\times\left(\frac{x}{m(1-\lambda)}\right)^{\tau+\frac{v}{k}+\mu\nu+1} \times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\Bigg| -\frac{x^{2\mu}}{4m^{2\mu}(1-\lambda)^{2\mu}};p,q\right]\\ &\times\sum_{r=0}^{\infty}\frac{\Gamma(r+1)\Gamma(1+\xi r)}{\Gamma\left(r+\frac{v}{k}+\frac{3}{2}\right)\Gamma\left(r+\frac{3}{2}\right)}\frac{\Gamma\left(\tau+2r+2-2n\mu-\mu\nu\right)}{\Gamma\left(1+\frac{\zeta}{1-\lambda}+\tau-2n\mu-\mu\nu+2r+2\right)r!}\\ &\times\left(\frac{-cx^{2+\xi}}{4kg(\sigma)^{\xi}m^2(1-\lambda)^2}\right)^{r} \end{aligned} \tag{3.6}$$

Using the definition of (1.22) in Eq.(3.6), we at once arrive the desired result in (3.1).

Theorem 3.2. *Let the parametric constraints $\zeta,\tau,v,c\in\mathbb{C}$ and $\lambda<1,|\frac{1}{t}|\leq 1$ be such that $\Re(\zeta)>0,\Re(\tau)>0,v>\frac{-3k}{2}$ and $\Re(\frac{\tau}{1-\lambda})>-1$. The above condition ensures the convergence of the series representation and guarantees the validity of the obtained integral transform,then the below mentioned Jafari transformation holds true:*

$$\begin{aligned}&T\left[P_{0_+}^{(\zeta,\lambda)}\{t^{\tau-1}J_{\nu,p,q}(\sigma t^{-\mu})\left(1-\cos\left(\frac{\varpi t^{\xi}z^{\xi}}{\sqrt{k}}\right)\right)\}(x)\right] = \frac{h(\sigma)}{g(\sigma)}x^{\zeta}\frac{\varpi\sqrt{\pi}}{4sk^2\Gamma(\nu+1)}\times\left(\frac{\sigma}{2}\right)^{\nu}\\ &\times\left(\frac{x}{[m(1-\lambda)]}\right)^{\tau-\mu\nu+2}\times\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\times{}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\Bigg| -\frac{\sigma^2m^{2\mu}(1-\lambda)^{2\mu}}{4x^{2\mu}};p,q\right]\\ &\times{}_3\psi_3\left[\begin{array}{ccc}(\tau-2n\mu-\mu\nu+2,2),(1,1),(1,\xi) & ; & \\ (\tau-2n\mu+\mu\nu-\frac{\zeta}{1-\lambda}+3,2),(2,1),(\frac{3}{2},1) & ; & \end{array} -\frac{\varpi^2x^{2+\xi}}{4kg(\sigma)^{\xi}m^2(1-\lambda)^2}\right]\end{aligned} \tag{3.7}$$

Theorem 3.3. *Let the parametric constraints* $\zeta, \tau, v, c \in \mathbb{C}, h(\sigma)$ *and* $g(\sigma)$ *are positive real numbers but* $(\sigma) \neq 0$ *and* $\lambda < 1, |\frac{1}{t}| \leq 1$ *be such that* $\Re(\zeta) > 0, \Re(\tau) > 0, v > \frac{-3k}{2}$ *and* $\Re(\frac{\tau}{1-\lambda}) > -1$. *The above condition ensures the convergence of the series representation and guarantees the validity of the obtained integral transform, then the below mentioned Jafari transformation holds true:*

$$T\left[P_{0_+}^{(\zeta,\lambda)}\{t^{\tau-1}J_{\nu,p,q}(\sigma t^{-\mu})\left(\cosh\left(\frac{\varpi t^{\xi}z^{\xi}}{\sqrt{k}}-1\right)\right)\}(x)\right] = \frac{h(\sigma)}{g(\sigma)}x^{\zeta}\frac{\varpi\sqrt{\pi}}{4sk^2\Gamma(\nu+1)}\times\left(\frac{\sigma}{2}\right)^{\nu}$$

$$\times\left(\frac{x}{[m(1-\lambda)]}\right)^{\tau-\mu\nu+2}\times\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\times{}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle|-\frac{\sigma^2m^{2\mu}(1-\lambda)^{2\mu}}{4x^{2\mu}};p,q\right]$$

$$\times{}_3\psi_3\left[\begin{array}{cc}(\tau-2n\mu-\mu\nu+2,2),(1,1),(1,\xi) & ; \\ (\tau-2n\mu+\mu\nu-\frac{\zeta}{1-\lambda}+3,2),(2,1),(\frac{3}{2},1) & ;\end{array}\frac{\varpi^2x^{2+\xi}}{4kg(\sigma)^{\xi}m^2(1-\lambda)^2}\right] \quad (3.8)$$

Theorem 3.4. *Let the parametric constraints* $\zeta, \tau, v, c \in \mathbb{C}, h(\sigma)$ *and* $g(\sigma)$ *are positive real numbers but* $(\sigma) \neq 0$ *and* $\lambda < 1, |\frac{1}{t}| \leq 1$ *be such that* $\Re(\zeta) > 0, \Re(\tau) > 0, v > \frac{-3k}{2}$ *and* $\Re(\frac{\tau}{1-\lambda}) > -1$. *The above condition ensures the convergence of the series representation and guarantees the validity of the obtained integral transform, then the below mentioned Jafafi transformation holds true:*

$$T\left[P_{0_+}^{(\zeta,\lambda)}\{t^{\tau-1}J_{\nu,p,q}(\sigma t^{-\mu})sin\left(\frac{\varpi t^{\xi}z^{\xi}}{\sqrt{k}}\right)\}(x):h,g\right] = \frac{h(\sigma)}{g(\sigma)}x^{\zeta}\times\frac{\varpi}{2s\Gamma(\nu+1)}\sqrt{\frac{\pi}{k}}\left(\frac{\sigma}{2}\right)^{\nu}$$

$$\times\left(\frac{x}{[m(1-\lambda)]}\right)^{\tau-\mu\nu+1}\times\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\times{}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle|-\frac{\sigma^2m^{2\mu}(1-\lambda)^{2\mu}}{4x^{2\mu}};p,q\right]$$

$$\times{}_2\psi_2\left[\begin{array}{c}(\tau-2n\mu-\mu\nu+1,2),(1,\xi); \\ (\tau-2n\mu-\mu\nu-\frac{\zeta}{1-\lambda}+2,2),(\frac{3}{2},1),;\end{array}\quad-\frac{\varpi^2x^{2+\xi}}{4kg(\sigma)^{\xi}m^2(1-\lambda)^2}\right] \quad (3.9)$$

Theorem 3.5. *Let the parametric constraints* $\zeta, \tau, v, c \in \mathbb{C}, h(\sigma)$ *and* $g(\sigma)$ *are positive real numbers but* $(\sigma) \neq 0$ *and* $\lambda < 1, |\frac{1}{t}| \leq 1$ *be such that* $\Re(\zeta) > 0, \Re(\tau) > 0, v > \frac{-3k}{2}$ *and* $\Re(\frac{\tau}{1-\lambda}) > -1$. *The above condition ensures the convergence of the series representation and guarantees the validity of the obtained integral transform, then the below mentioned Jafari transformation holds true:*

$$T\left[P_{0_+}^{(\zeta,\lambda)}\{t^{\tau-1}J_{\nu,p,q}(\sigma t^{-\mu})sinh\left(\frac{\varpi t^{\xi}z^{\xi}}{\sqrt{k}}\right)\}(x)\right] = \frac{h(\sigma)}{g(\sigma)}x^{\zeta}\times\frac{\varpi}{2s\Gamma(\nu+1)}\sqrt{\frac{\pi}{k}}\left(\frac{\sigma}{2}\right)^{\nu}$$

$$\times\left(\frac{x}{[m(1-\lambda)]}\right)^{\tau-\mu\nu+1}\times\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\times{}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle|-\frac{\sigma^2m^{2\mu}(1-\lambda)^{2\mu}}{4x^{2\mu}};p,q\right]$$

$$\times {}_2\psi_2\left[\begin{array}{lll}(\tau-2n\mu-\mu\nu+1,2),(1,\xi) & ; & \dfrac{\varpi^2x^{2+\xi}}{4kg(\sigma)^{\xi}m^2(1-\lambda)^2}\\ (\tau-2n\mu-\mu\nu-\frac{\zeta}{1-\lambda}+2,2),(\frac{3}{2},1) & ; & \end{array}\right] \quad (3.10)$$

The proof of Theorem 3.2, 3.3, 3.4 and 3.5 are developed following the similar lines as to prove the Theorem 3.1 respectively in view of the definitions of generalized k-struve function , (p,q)-Extended Bessel function and Jafari transform defined in (1.7),(1.17) and (1.23).

4. Applications

In this section, we obtained new and known results.

Corollary 4.1. *If we put $k=1$ and $c=1$ in Theorem* (2.1)*, then it reduces to the following Struve function of order v, so we get the following results:*

$$\left(P_{0_+}^{(\zeta,\lambda)}\left[t^{\tau-1}J_{\nu,p,q}(t^{\mu})H_v(t)\right]\right)(x)=\frac{x^{\zeta}}{\Gamma(\nu+1)}\times\frac{1}{2^{v+1+\nu}}\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)$$

$$\times\left(\frac{x}{m(1-\lambda)}\right)^{\tau+v+\mu\nu+1}\times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle|-\frac{x^{2\mu}}{4m^{2\mu}(1-\lambda)^{2\mu}};p,q\right]$$

$$\times {}_2\psi_3\left[\begin{array}{lll}(\tau+v+2n\mu+\mu\nu+1,2),(1,1) & ; & -\dfrac{x^2}{4m^2(1-\lambda)^2}\\ (\tau+v+2n\mu+\mu\nu+\frac{\zeta}{1-\lambda}+2,2),(v+\frac{3}{2},1),(\frac{3}{2},1) & ; & \end{array}\right] \quad (4.1)$$

Corollary 4.2. *If we put $k=1$ in Theorem* (2.2)*, then it reduces to the following classical Struve function of order v, so we get the following results:*

$$\left(P_{0_+}^{(\zeta,\lambda)}\left[t^{\tau-1}J_{\nu,p,q}(\sigma t^{-\mu})\left(1-\cos\left(\varpi t\right)\right)\right]\right)(x)=x^{\zeta}\frac{\varpi\sqrt{\pi}}{4\Gamma(\nu+1)}\times\left(\frac{\sigma}{2}\right)^{\nu}$$

$$\times\left(\frac{x}{[m(1-\lambda)]}\right)^{\tau-\mu\nu+2}\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle|-\frac{\sigma^2m^{2\mu}(1-\lambda)^{2\mu}}{4x^{2\mu}};p,q\right]$$

$$\times {}_2\psi_3\left[\begin{array}{lll}(\tau-2n\mu-\mu\nu+2,2),(1,1) & ; & -\dfrac{\varpi^2x^2}{4m^2(1-\lambda)^2}\\ (\tau-2n\mu+\mu\nu-\frac{\zeta}{1-\lambda}+3,2),(2,1),(\frac{3}{2},1) & ; & \end{array}\right] \quad (4.2)$$

Corollary 4.3. *If we put $k=1$ in Theorem* (2.4)*, then it reduces to the following classical Struve function of order v, so we get the following results:*

$$\left(P_{0_+}^{(\zeta,\lambda)}\left[t^{\tau-1}J_{\nu,p,q}(\sigma t^{-\mu})sin\left(\varpi t\right)\right]\right)(x)=x^{\zeta}\times\frac{\varpi}{2\Gamma(\nu+1)}\sqrt{\pi}\left(\frac{\sigma}{2}\right)^{\nu}$$

$$\times\left(\frac{x}{[m(1-\lambda)]}\right)^{\tau-\mu\nu+1}\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle|-\frac{\sigma^2m^{2\mu}(1-\lambda)^{2\mu}}{4x^{2\mu}};p,q\right]$$

$$\times {}_1\psi_2\left[\begin{array}{ccc} (\tau-2n\mu-\mu\nu+1,2) & ; & -\dfrac{\varpi^2x^2}{km^2(1-\lambda)^2} \\ (\tau-2n\mu-\mu\nu-\frac{\zeta}{1-\lambda}+2,2),(\frac{3}{2},1) & ; & \end{array}\right] \tag{4.3}$$

Corollary 4.4. *If we put $k=1$ and $c=1$ in Theorem (3.1), then it reduces to the following Struve function of order v, so we get the following results:*

$$T\left[P_{0_+}^{(\zeta,\lambda)}\{t^{\tau-1}J_{\nu,p,q}(t^{\mu})H_v(tz)^{\xi}\}(x)\right]=\frac{h(\sigma)}{g(\sigma)}\frac{x^{\zeta}}{\Gamma(\nu+1)}\times\frac{1}{2^{\frac{v}{k}+1+\nu}}$$

$$\Gamma\left(1+\frac{\zeta}{1-\lambda}\right)\times\left(\frac{x}{m(1-\lambda)}\right)^{\tau+v+\mu\nu+1}\times {}_1F_2\left[\begin{array}{c}\frac{1}{2}\\ \frac{1}{2},\nu+1\end{array}\middle|-\frac{x^{2\mu}}{4m^{2\mu}(1-\lambda)^{2\mu}};p,q\right]$$

$$\times {}_3\psi_3\left[\begin{array}{ccc} (\tau+v+2n\mu+\mu\nu+1,2),(1,1),(1,\xi) & ; & -\dfrac{x^{2+\xi}}{4g(\sigma)^{\xi}m^2(1-\lambda)^2} \\ (\tau+v+2n\mu+\mu\nu+\frac{\zeta}{1-\lambda}+2,2),(v+\frac{3}{2},1),(\frac{3}{2},1) & ; & \end{array}\right] \tag{4.4}$$

Other known results are below:

- If we consider $J_{\nu,p,q}(t^{\mu})=1$ and $\rho=\tau,\eta=\zeta,\alpha=\lambda$ and $a=m$. Theorem (2.1) reduces to known result due to Kottakkaran Sooppy Nisar et.al. ([23], pp.1269, eq. (8)).

- If we consider $J_{\nu,p,q}(t^{-\mu})=1,\sigma=1$ and $\rho=\tau,\eta=\zeta,\alpha=\lambda,\gamma=\varpi$ and $m=a$ Theorem (2.2) reduces to known result due to Kottakkaran Sooppy Nisar et.al.([23], pp.1271, eq. (14)).

- If we consider $J_{\nu,p,q}(t^{-\mu})=1,\sigma=1$ and $\rho=\tau,\eta=\zeta,\alpha=\lambda,\gamma=\varpi$ and $m=a$.Theorem (2.3) reduces to known result due to Kottakkaran Sooppy Nisar et.al.([23], pp.1272, eq. (16)).

- If we consider $J_{\nu,p,q}(t^{-\mu})=1,\sigma=1$ and $\rho=\tau,\eta=\zeta,\alpha=\lambda,\gamma=\varpi$ and $m=a$ .Theorem (2.4) reduces to known result due to Kottakkaran Sooppy Nisar et.al.([23], pp.1272, eq. (18)).

- If we consider $J_{\nu,p,q}(t^{-\mu})=1,\sigma=1$ and $\rho=\tau,\eta=\zeta,\alpha=\lambda,\gamma=\varpi$ and $m=a$.Theorem (2.5) reduces to known result due to Kottakkaran Sooppy Nisar et.al.([23], pp.1273, eq. (20)).

5. Conclusion

In present paper, our main objective was to apply the Jafari integral transforms to a Pathway fractional integral operator involving the product of the (p,q)-Extended Bessel function and Generalized k-Struve function. The importance of the study lies in the applications shown in Section-3. As in our present investigation we have introduced and studied a Jafari transform. All the theorems and corollaries of Section 2, 3 and 4 can be results for generalized k-Wright function and (p, q) -extended Generalized hypergeometric function. We can simply obtain various known and new results in applications.

Acknowledgement

The author confirms being the sole contributions of this work and approved it publications.

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